We first find the ACF.

\[ \gamma(0) = \text{Var}(w_t + w_{t-2}) = \text{Var}(w_t) + \text{Var}(w_{t-2}) = 2 \]

\[ \gamma(1) = \text{Cov}(w_t + w_{t-2}, w_{t-1} + w_{t-3}) = 0 \]

\[ \gamma(2) = \text{Cov}(w_t + w_{t-2}, w_{t-2} + w_{t-4}) = 1 \]

\[ \gamma(h) = \begin{cases} 
2, & h=0 \\
1, & h = \pm 2 \\
0, & \text{otherwise}.
\end{cases} \]

\[ \phi_{11} : \quad \phi_{11} = \gamma(1) = \frac{0}{2} = 0 \]

\[ \phi_{22} : \quad \text{Look at } \hat{\chi}_3^2 = \chi_1 X_1 + \chi_2 X_2. \]

\[ \text{E}[(X_7 - \chi_1 X_1 - \chi_2 X_2) X_1] = 0 \]

\[ \frac{\gamma(1)}{2} - \chi_1 \delta(0) - \chi_2 \gamma(1) = 0 \]

\[ 1 - \chi_1 \gamma(1) - \chi_2 \gamma(0) = 0 \quad \Rightarrow \quad \chi_1 = \gamma_2 \]

\[ \boxed{\phi_{22} = \frac{1}{2}} \]
We first find the ACF.

Since $X_t = 0.8X_{t-1} + W_t$, 

$$X_t = 0.8(0.8X_{t-2} + W_{t-1}) + W_t$$

$$= W_t + 0.8W_{t-1} + 0.64X_{t-2}$$

$$= \ldots = W_t + 0.8W_{t-1} + 0.8^2W_{t-2} + \ldots$$

If $h=0$, then $\delta_h(0) = V(X_t) =$

$$\sum_{h=0}^{\infty} 0.8^2^h = \frac{1}{1-0.8^2} = \frac{1}{0.36}.$$  

(continued)
(2) (continued)

If \( h \) is odd, \( \gamma(h) = 0 \) since \( x_t \) and \( x_{t+h} \) have no \( w_t \) values in common.

If \( h > 0 \) and \( h \) is even, then \( \gamma_x(h) \)
\[
= \text{cov}(x_t, x_{t+h}) = \text{cov}(w_t + 0.8 w_{t-1} + \ldots, w_{t+h} + 0.8 w_{t+h-1} + \ldots)
\]
\[
= 1 - (0.8)^{h/2} + 0.8 \left( (0.8)^{h/2} \right) + \ldots = \frac{0.8^{h/2}}{1 - 0.8^{2}}
\]
from \( w_t \) from \( w_{t-1} \).

\[
\gamma_x(h) = \begin{cases} 
\frac{0.8^{h/2}}{0.36}, & h \text{ even}, \\
0, & h \text{ odd}.
\end{cases}
\]

\[
\beta_x(h) = \begin{cases} 
0.8^{h/2}, & h \text{ even}, \\
0, & h \text{ odd}.
\end{cases}
\]

\( \Phi_{11} : \) By definition, \( \Phi_{11} = \beta_x(1) = 0.8. \)

\( \Phi_{22} : \) Since the process is AR(2), \( \Phi_{22} = \Phi_2 = 0.8. \)

Also, since the process is AR(2), \( \Phi_{hh} = 0 \) for \( h > 2 \).

\[
\Phi_{hh} = \begin{cases} 
0.8, & h=2, \\
0, & h=3,4,\ldots
\end{cases}
\]
See my R code for full details.

I first plotted the data. I saw no sign of non-stationarity.

I then plotted the ACF and the PACF.

The sample PACF seemed to cut off after lag 1 or lag 2, suggesting that an AR(1) or AR(2) model might work well.

I checked AIC values for ARMA(p,q) models with $0 \leq p \leq 4$ and $0 \leq q \leq 4$. The lowest AIC values were for AR(2) + ARMA(1,1).

I fit both AR(2) + ARMA(1,1) models. In both cases, the diagnostics looked good. Both the sample ACF and the Ljung-Box test p-values suggested that the standardized residuals were consistent with white noise. In the end, I chose the AR(2) model for its slightly smaller AIC.

Fitted model:

\[
\hat{X}_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2)
\]

For 2016: \[\hat{X}_t = 0.423\]
(4) Since the ARVF cuts off after lag 2, the process must be a MA(2) process
\[ X_t = \omega_0 + \omega_1 w_{t-1} + \omega_2 w_{t-2} \text{ for} \]
\[ w_t \sim \mathcal{N}(0, \sigma^2). \]

Then \( (1 + \omega_1^2 + \omega_2^2) \sigma^2 = 14, \)
\( (\omega_1 + \omega_2) \sigma = -4, \) and
\( \omega_2 \sigma = -3. \)

Since \( 14 + 2(-4) + 2(-3) = 0, \)
\( (1 + \omega_1^2 + \omega_2^2 + 2\omega_1 + 2\omega_2 \omega_1 + 2\omega_2)^2 = 0 \)

\( \Rightarrow (1 + \omega_1 + \omega_2)^2 = 0 \Rightarrow 1 + \omega_1 + \omega_2 = 0. \)

Thus \( \omega_1 = -1 - \omega_2. \) Dividing \( \omega_1 + \omega_2 = \omega \) gives
\[ \frac{\omega_1 + \omega_2}{\omega_2} = \frac{4}{3} \Rightarrow \frac{\omega_1 (\omega_1 + \omega_2)}{\omega_2} = \frac{4}{3} \]

\( \Rightarrow \frac{(-1 - \omega_2)(1 + \omega_2)}{\omega_2} = \frac{4}{3} \)
\[ \Rightarrow \frac{(1 + \omega_2)^2}{\omega_2} = \frac{4}{3} \Rightarrow -3(1 + 2\omega_2 + \omega_2^2) = 4\omega_2 \]
\[ \Rightarrow 3\omega_2^2 + 4\omega_2 + 3 = 0 \]
\[ \Rightarrow (3\omega_2 + 1)(\omega_2 + 3) = 0 \]
(continued)
4. (continued)

\[ \theta_2 = -\frac{1}{3} \text{ or } \theta_2 = -3. \]

If \( \theta_2 = -\frac{1}{3} \), then \( \theta_1 = -1 - \theta_2 = -1 + \frac{1}{3} = -\frac{2}{3}. \)

Also, by (ii), \( \sigma^2 \theta_2 = \frac{-3}{-\frac{2}{3}} = \frac{-3}{-\frac{2}{3}} = 9. \)

**Solution #1:** \[ X_t = w_t - \frac{2}{3} w_{t-1} - \frac{1}{3} w_{t-2}, \text{ where } w_t \sim WN(0, 9). \]

If \( \theta_2 = -3 \) then \( \theta_1 = -1 + 3 = 2. \) Also, by

\[ \sigma^2 \theta_2 = \frac{-3}{-3} = 1. \]

**Solution #2:** \[ X_t = w_t + 2w_{t-1} - 3w_{t-2}, \text{ where } w_t \sim WN(0, 1). \]

5. Based on our solution to 4, we see that 2 possible solutions are

\#1: \[ X_t = w_t - \frac{2}{3} w_{t-1} - \frac{1}{3} w_{t-2}, \text{ where } w_t \sim WN(0, 9) \text{ and } \]

\#2: \[ X_t = w_t + 2w_{t-1} - 3w_{t-2}, \text{ where } w_t \sim WN(0, 1). \]