1. (To turn in.) Let $x_t$ be a stationary time series with mean $\mu$ and ACF $\rho(\cdot)$, and let $h > 0$. Show that the best predictor of $X_{n+h}$ of the form $aX_n + b$ is obtained by choosing $a = \rho(h)$ and $b = \mu(1 - \rho(h))$.

2. (To turn in.) Consider the MA(2) process $x_t = w_t + w_{t-1} + w_{t-2}$, where $w_t$ is a white noise process with mean 0 and variance 1. (a) Find the ACVF for $x_t$. (b) Find the predictors $x_2^1$ and $x_3^2$. (c) Suppose that we wish to predict $x_2$ from $x_1$ and $x_3$. Find the best predictor of $x_2$ of the form $\hat{x}_2 = a_1x_1 + a_3x_3$, where $a_1$ and $a_3$ are constants. (d) Suppose that we wish to predict $x_3$ from $x_1$ and $x_5$. Find the best predictor of $x_3$ of the form $\hat{x}_3 = a_1x_1 + a_5x_5$, where $a_1$ and $a_5$ are constants. (e) Find the mean squared prediction error (MSPE) for the predictor in (d).

3. (Not to turn in) Consider the time series $x_t = A\cos(\omega t) + B\sin(\omega t)$, where $A$ and $B$ are uncorrelated random variables with mean 0 and variance 1 and $\omega$ is a fixed constant. (a) Show that $x_t$ is stationary and find its mean and ACVF. (b) Find the predictors $x_2^3$ and $x_3^2$. (c) Find the MSPE for each of the predictors from (b).

4. (To turn in) Consider the function $\gamma(h)$ defined by

$$\gamma(h) = \begin{cases} 1, & h = 0, \\ -0.5, & h = \pm 2, \text{ and} \\ 0, & \text{otherwise}. \end{cases}$$

(a) Show that $\gamma$ is an ACVF by providing a stationary process $x_t$ that has $\gamma$ as its ACVF. (b) For the process $x_t$ with ACVF $\gamma$, find the predictor $x_3^2$. (c) What is the MSPE for the predictor $x_3^2$ that you found in part (b)?