1. Find a filter of the form $1 + \alpha B + \beta B^2 + \gamma B^3$ that passes linear trends without distortion and that eliminates arbitrary seasonal components of period 2. That is, find real constants $\alpha$, $\beta$, and $\gamma$ so that time series of the form $x_t = a + bt$ pass through the filter unchanged and so that time series of the form $s_t, -s_t, s_t, -s_t, \ldots$ are zeroed out.

2. Let $\{y_t\}$ be a stationary time series with mean 0, and let $a$ and $b$ be constants. Set $x_t = a + bt + s_t + y_t$, where $s_t$ is a seasonal component with period 12. Show that the time series $\nabla x_t = (1 - B)(1 - B^{12})x_t$ is stationary, and express its ACVF in terms of the ACVF for $\{y_t\}$.

(Hint: Recall that $B$ is the backward shift operator $Bx_t = x_{t-1}$.)

3. (Not to turn in.) Show that the filter that maps $x_t$ to $\frac{1}{9}(-x_{t+2} + 4x_{t+1} + 3x_t + 4x_{t-1} - x_{t-2})$ passes third-degree polynomials and eliminates seasonal components with period 3.

4. (Not to turn in.) Show that if $\{x_t\}$ and $\{y_t\}$ are uncorrelated stationary time series, then $\{x_t + y_t\}$ is also a stationary time series. Express the ACVF for $\{x_t + y_t\}$ in terms of $\gamma_x(t)$ and $\gamma_y(t)$.

Minitab assignment:

Using Minitab, apply the classical decomposition to the airline passengers dataset that is available on the course website. Instructions for accessing Minitab, which is available through the university, are also available on the course website. Complete each of the six parts and turn in the following four plots: (1) time series plot from part 1, (2) sample ACF plot from part 2, (3) time series plot from part 4, and (4) time series decomposition plot (with the forecasts) from part 5. Please also turn in your comments from part 6.


2. Plot the sample ACF for the raw data using Stat - Time Series - Autocorrelation.

3. Transform the raw data into a new dataset by taking logarithms using Calc - Calculator. Store the new data in a new column.

4. Make a time series plot of the new data.

5. Implement the classical decomposition for the new data using Stat - Time Series - Decomposition. Choose seasonal length 12, and indicate that you wish to fit an additive model. Include both trend and seasonal components, and generate forecasts for 12 periods.

6. How reasonable do the forecasts seem? Do you see any places where the model doesn’t seem to fit well? Please explain.